

TRIANGLE TOOLKIT

OPP

ADJ

TAN

Tool Name: **Tangent Ratio**

When can we use this?

Right Δ only!

Involves: an acute angle

$$\tan(45) = \frac{1}{1}$$

its **Opposite Leg**
and its **Adjacent Leg**

$$\frac{\tan(<)}{1} = \frac{\text{Opposite Leg}}{\text{Adjacent Leg}}$$

$$\frac{\tan(<)}{1} = \frac{\text{rise}}{\text{run}} \text{ or } \frac{\Delta y}{\Delta x}$$

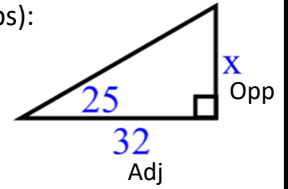
$$\tan(<) = \text{Slope}$$

Example (must show all steps):

$$\frac{\tan(25)}{1} = \frac{x}{32}$$

$$32 \cdot \tan(25) = x$$

$$14.92 \approx x$$



OPP

HYP

SIN

Tool Name: **Sine Ratio**

When can we use this?

Right Δ only!

Involves: an acute angle

$$\sin(60) = \frac{\sqrt{3}}{2}$$

its **Opposite Leg**
and the **Hypotenuse**

$$\frac{\sin(<)}{1} = \frac{\text{Opposite Leg}}{\text{Hypotenuse}}$$

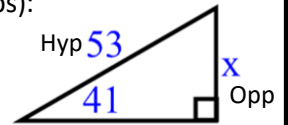
$$\frac{\sin(<)}{1} = \frac{\text{rise}}{\text{hyp}} \text{ or } \frac{\Delta y}{\text{hyp}}$$

Example (must show all steps):

$$\frac{\sin(41)}{1} = \frac{x}{53}$$

$$53 \cdot \sin(41) = x$$

$$34.77 \approx x$$



ADJ

HYP

COS

Tool Name: **Cosine Ratio**

When can we use this?

Right Δ only!

Involves: an acute angle

$$\cos(60) = \frac{1}{2}$$

its **Adjacent Leg**
and the **Hypotenuse**

$$\frac{\cos(<)}{1} = \frac{\text{Adjacent Leg}}{\text{Hypotenuse}}$$

$$\frac{\cos(<)}{1} = \frac{\text{run}}{\text{hyp}} \text{ or } \frac{\Delta x}{\text{hyp}}$$

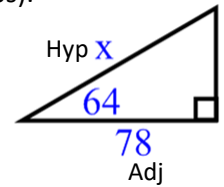
Example (must show all steps):

$$\frac{\cos(64)}{1} = \frac{78}{x}$$

$$x \cdot \cos(64) = 78$$

$$x = \frac{78}{\cos(64)}$$

$$x \approx 177.93$$



HYP

IS

C!

Tool Name: **Pythagorean Theorem**

When can we use this?

Right Δ only!

Involves: Angle measures ok, but not needed

Must have: **2 side lengths**

Trying to find: **3rd side length**

$$\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$$

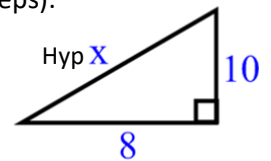
$$a^2 + b^2 = c^2$$

Example 1 (must show all steps):

$$8^2 + 10^2 = x^2$$

$$164 = x^2$$

$$\sqrt{164} = x$$



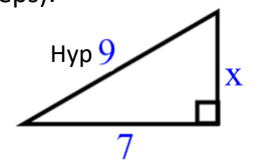
Example 2 (must show all steps):

$$x^2 + 7^2 = 9^2$$

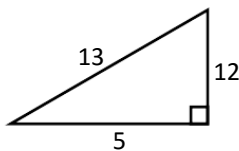
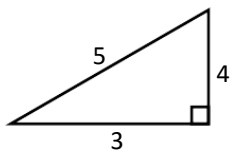
$$x^2 + 49 = 81$$

$$\frac{-49 \quad -49}{x^2 = 32}$$

$$x = \sqrt{32}$$



Common triples:



**FIND
THE
ANGLE**

Tool Name: **Inverse Trig**

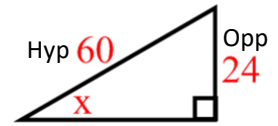
Remember!

Example (must show all steps):

When can we use this?

$$\frac{\tan(\angle)}{1} = \frac{Opp}{Adj} \quad \frac{\sin(\angle)}{1} = \frac{Opp}{Hyp}$$

$$\frac{\sin(x)}{1} = \frac{24}{60}$$



Right Δ only!

Involves: Must have: **2 side lengths**

$$\frac{\cos(\angle)}{1} = \frac{Adj}{Hyp}$$

$$x = \sin^{-1}\left(\frac{24}{60}\right)$$

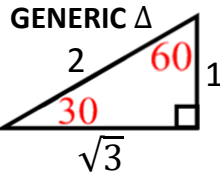
Trying to find: **one acute angle**

$$23.58 \approx x$$

Can be: **\tan^{-1} or \sin^{-1} or \cos^{-1}**

30-60

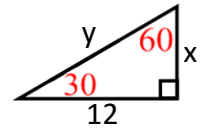
Tool Name: **30-60-90 Δ**



Just multiply the sides of the **GENERIC by the zoom factor**

When can we use this?

$$zf = \frac{12 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{9}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$



30-60-90 Δ only!

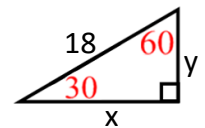
$$\text{so } x = 4\sqrt{3} \cdot 1 = 4\sqrt{3}, \text{ and } y = 4\sqrt{3} \cdot 2 = 8\sqrt{3}$$

Must have: **a 30-60-90 Δ**

Can't assume: **Angle measures**

In the **generic Δ**, $\sqrt{3}$ is **ALWAYS** across from the 60° angle!

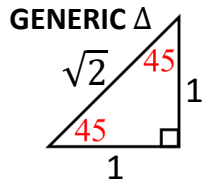
$$zf = \frac{18}{2} = 9, \text{ so } y = 9 \cdot 1 = 9,$$



$$\text{and } x = 9 \cdot \sqrt{3} = 9\sqrt{3}$$

45-45

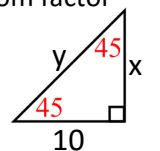
Tool Name: **45-45-90 Δ**



Just multiply the sides of the **GENERIC by the zoom factor**

When can we use this?

$$zf = \frac{10}{1} = 10, \text{ so } x = 10 \cdot 1 = 10,$$



45-45-90 Δ only!

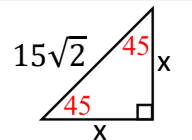
$$\text{and } y = 10 \cdot \sqrt{2} = 10\sqrt{2}$$

Must have: **a 45-45-90 Δ**

Can't assume: **Angle measures**

In the **generic Δ**, $\sqrt{2}$ is **ALWAYS** the hypotenuse!

$$zf = \frac{15\sqrt{2}}{\sqrt{2}} = 15,$$



$$\text{so } x = 15 \cdot 1 = 15$$

**NON
RIGHT
Δ's!**

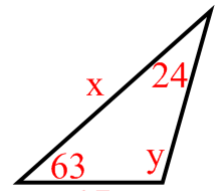
Tool Name: **Law of Sines**

Can be used on a non-right Δ!

Example (must show all steps):

When can we use this?

$$\frac{\sin(24)}{17} = \frac{\sin(93)}{x}$$



Must have: **2 angles and one opposite side**

$$x \cdot \sin(24) = 17 \cdot \sin(93)$$

Trying to find: **other opposite side**

OR

$$x = \frac{17 \cdot \sin(93)}{\sin(24)}$$

Must have: **2 sides and one opposite angle**

$$x \approx 41.74$$

$$y = 93 \text{ (Δ sum)}$$

Trying to find: **other opposite angle**

**NON
RIGHT
Δ's!
LOOK
FOR
SAS!**

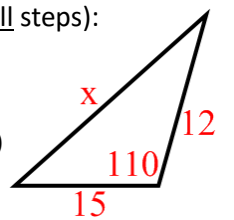
Tool Name: **Law of Cosines**

Can be used on a non-right Δ!

Example (must show all steps):

When can we use this?

$$x^2 = 15^2 + 12^2 - 2 \cdot 15 \cdot 12 \cdot \cos(110)$$



Must have: **SAS (side-angle-side)**

$$x^2 = 369 - 360 \cdot \cos(110)$$

Trying to find: **3rd side**

$$x = \sqrt{369 - 360 \cdot \cos(110)}$$

OR

Must have: **3 sides**

$$x \approx 22.18$$

Trying to find: **one angle**