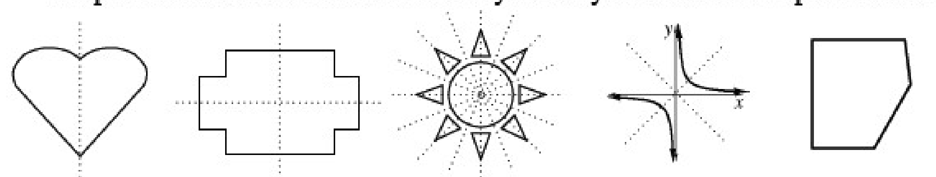
MATH NOTES-Chapters 1-6

MATH NOTES

METHODS AND MEANINGS

Lines of Symmetry

When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **reflective symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



This shape has one line of symmetry.	This shape has two lines of symmetry.	This shape has eight lines of symmetry.	This graph has two lines of symmetry.	This shape has no lines of symmetry.

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The Investigative Process

The **investigative process** is a way to study and learn new mathematical ideas. Mathematicians have used this process for many years to make sense of new concepts and to broaden their understanding of older ideas.

Question → Explore and Conjecture → Prove

In general, this process begins with a **question** that helps you frame what you are looking for. For example, a question such as, "*What if the Möbius strip has 2 half-twists? What will happen when that strip is cut in half down the middle?*" can help start an investigation to find out what happens when the Möbius strip is slightly altered.

Once a question is asked, you can make an educated guess, called a **conjecture**. This is a mathematical statement that has not yet been proven.

Next, **exploration** begins. This part of the process may last awhile as you gather more information about the mathematical concept. For example, you may first have an idea about the diagonals of a rectangle, but as you draw and measure a rectangle on graph paper, you find out that your conjecture was incorrect. When this happens, you just experiment some more until you have a new conjecture to test.

When a conjecture seems to be true, the final step is to **prove** that the conjecture is always true. A proof is a convincing logical argument that uses definitions and previously proven conjectures in an organized sequence.

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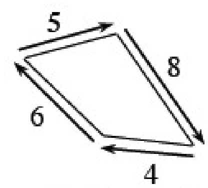
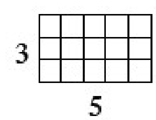
The Perimeter and Area of a Figure

The **perimeter** of a two-dimensional figure is the distance around its exterior (outside) on a flat surface. It is the total length of the boundary that encloses the interior (inside) region. See the example at right.



Perimeter = $5 + 8 + 4 + 6 = 23$ units

The **area** indicates the number of square units needed to fill up a region on a flat surface. For a rectangle, the area is computed by multiplying its length and width. The rectangle at right has a length of 5 units and a width of 3 units, so the area of the rectangle is 15 square units.



Area = $5 \cdot 3 = 15$ square units

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Solving Linear Equations

In Algebra, you learned how to solve a linear equation. This course will help you apply your algebra skills to solve geometric problems. Review how to solve equations by reading the example below.

- **Simplify.** Combine like terms on each side of the equation whenever possible.
- **Keep equations balanced.** The equal sign in an equation tells you that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.

$3x-2+4=x-6$	Combine like terms
$3x+2=x-6$	
$-x=-x$	Subtract x on both sides
$2x+2=-6$	
$-2=-2$	Subtract 2 on both sides
$\frac{2x}{2}=\frac{-8}{2}$	Divide both sides by 2
$x=-4$	

- **Move your x-terms to one side of the equation.** Isolate all variables on one side of the equation and the constants on the other.
- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x=-8$, since the 2 and the x are multiplied, then dividing both sides by 2 will get x alone.

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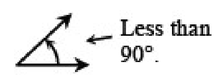
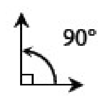
Types of Angles

When trying to describe shapes, it is convenient to classify types of angles. An angle is formed by two rays joined at a common endpoint. The measure of an angle represents the number of degrees of rotation from one ray to the other about the vertex. This course will use the following terms to refer to angles:

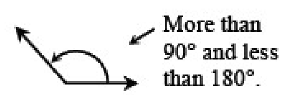
ACUTE: Any angle with measure *between* (but not including) 0° and 90°.



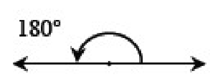
RIGHT: Any angle that measures 90°.

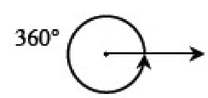


OBTUSE: Any angle with measure *between* (but not including) 90° and 180°.



STRAIGHT: Straight angles have a measure of 180° and are formed when the sides of the angle form a straight line.





CIRCULAR: Any angle that measures 360°.

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Probability Vocabulary and Definitions

Event: Any outcome, or set of outcomes, from a probabilistic situation. A **successful event** is the set of all outcomes that are of interest in a given situation. For example, rolling a die is a probabilistic situation. Rolling a 5 is an event. If you win a prize for rolling an even number, you can consider the set of three outcomes $\{2, 4, 6\}$ a successful event.

Sample space: All possible outcomes from a probabilistic situation. For example, the sample space for flipping a coin is heads and tails; rolling a die has a sample space of $\{1, 2, 3, 4, 5, 6\}$.

Probability: The likelihood that an event will occur. Probabilities may be written as ratios (fractions), decimals, or percents. An event that is certain to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that "might happen" have probabilities between 0 and 1, or between 0% and 100%. The more likely an event is to happen, the greater its probability.

Experimental probability: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical probability: Probability that is mathematically calculated. When each of the outcomes in the sample space has *equally likely chance* of occurring,

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

For example, to calculate the probability of rolling an even number on a die, first figure out how many possible (equally likely) outcomes there are. Since there are six faces on the number cube, the total number of possible outcomes is 6. Of the six faces, three of the faces are even numbers—there are three successful outcomes. Thus, to find the probability of rolling an even number, you would write:

$$P(\text{even}) = \frac{\#\text{ of successes}}{\text{total }\#\text{ possible}} = \frac{\text{number of ways to roll even numbers}}{\text{number of faces on die}} = \frac{1}{6} = 0.1\overline{6} \approx 16.7\%$$

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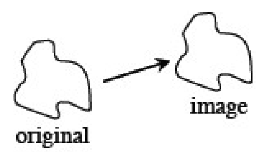
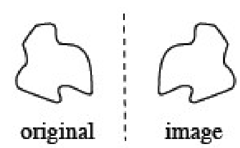
Rigid Transformations

A **rigid transformation** maps each point of a figure to a new point, so that the resulting image has the same size and shape of the original. There are three types of rigid transformations, described below.

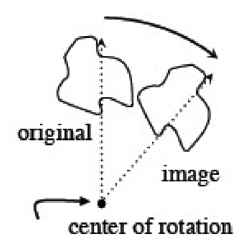
A transformation that preserves the size, shape, and orientation of a figure while *sliding* it to a new location is called a **translation**.



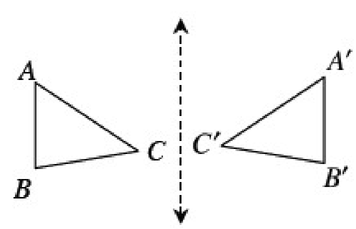
A transformation that preserves the size and shape of a figure across a line to form a mirror image is called a **reflection**. The mirror line is a **line of reflection**. One way to find a reflection is to *flip* a figure over the line of reflection.



A transformation that preserves the size and shape while *turning* an entire figure about a fixed point is called a **rotation**. Figures can be rotated either clockwise (↻) or counterclockwise (↺).



When labeling a transformation, the new figure (image) is often labeled with **prime notation**. For example, if ΔABC is reflected across the vertical dashed line, its image can be labeled $\Delta A'B'C'$ to show exactly how the new points correspond to the points in the original shape. We also say that ΔABC is **mapped** to $\Delta A'B'C'$.



METHODS AND MEANINGS

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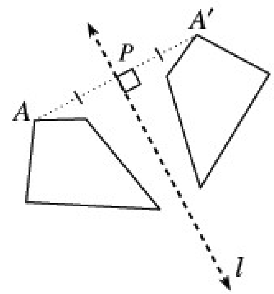
Formal Definitions of Rigid Transformations

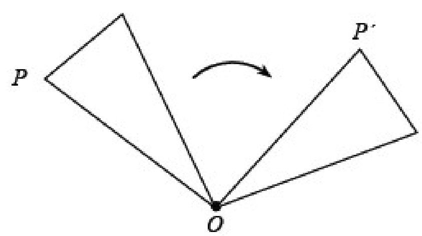
In algebra, you learned that a function is an equation that assigns each input a unique output. Most of these functions involved expressions and numbers such as $f(x) = 3x - 5$, so $f(2) = 1$.

In this course, you are now studying functions that assign each point in the plane to a unique point in the plane. These functions are called **rigid transformations (or motions)** because they move the entire plane with any figures you have drawn so that all of the figures remain unchanged. Therefore, angles and distances are preserved. There are three basic rigid motions that we will consider: reflections, translations, and rotations. All rigid motions can be seen as a combination of them.

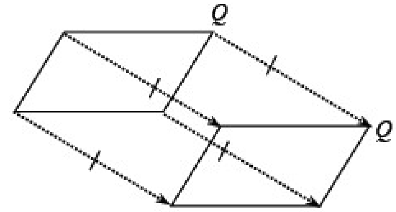
Reflections: When a figure is reflected across a line of reflection, such as the figure at right, it appears that the figure is "flipped" over the line. However, formally, a reflection across a line of reflection is defined as a function of each point (such as A) to a point (such as A') so that the line of reflection is the perpendicular bisector of the segments connecting the points and their images (such as $\overline{AA'}$). Therefore, $AP = A'P$.



Rotations: Formally, a rotation about a point O is a function that assigns each point (P) in the plane a unique point (P') so that all angles of rotation $\angle POP'$ have the same measure (which is the angle of rotation) and $OP = OP'$.



Translations: Formally, a translation is a function that assigns each point (Q) in the plane a unique point (Q') so that all line segments connecting an original point with its image have equal length and are parallel.



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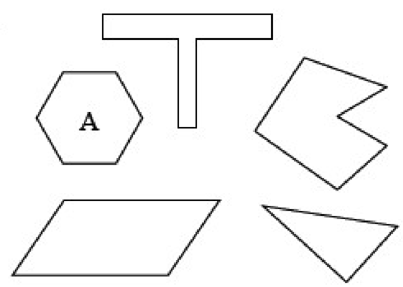
Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.



At right are some examples of polygons.

Shape A at right is an example of a **regular polygon** because its sides are all the same length and its angles have equal measure.

Polygons are named according to the number of sides that they have. Polygons that have 3 sides are **triangles**, those with 4 sides are **quadrilaterals**, polygons with 5 sides are **pentagons**, polygons with 6 sides are **hexagons**, polygons with 8 sides are **octagons**, polygons with 10 sides are **decagons**. For most other polygons, people simply name the number of sides, such as "11-gon" to indicate a polygon with 11 sides.

METHODS AND MEANINGS

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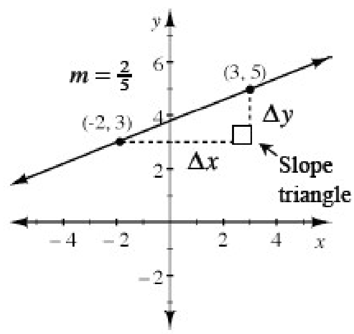
Slope of a Line and Parallel and Perpendicular Slopes

During this course, you will use your algebra tools to learn more about shapes. One of your algebraic tools that can be used to learn about the relationship of lines is slope. Review what you know about slope below.

The **slope** of a line is the ratio of the change in y (Δy) to the change in x (Δx) between any two points on the line. It indicates both how steep the line is and its direction, upward or downward, left to right.

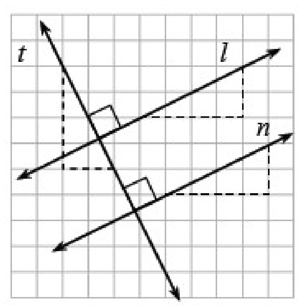
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$



Lines that point upward from left to right have positive slope, while lines that point downward from left to right have negative slope. A horizontal line has zero slope, while a vertical line has undefined slope. The slope of a line is denoted by the letter m when using the $y = mx + b$ equation of a line.

One way to calculate the slope of a line is to pick two points on the line, draw a slope triangle (as shown in the example above), determine Δy and Δx, and then write the slope ratio. Be sure to verify that your slope correctly resulted in a negative or positive value based on its direction.

Parallel lines are lines that lie in the same plane (a flat surface) and never intersect. Lines l and n at right are examples of parallel lines.



On the other hand, **perpendicular lines** are lines that intersect at a right angle. For example, lines t and n at right are perpendicular, as are lines t and l. Note that the small square drawn at the point of intersection indicates a right angle.

The **slopes of parallel lines** are the same. In general, the slope of a line parallel to a line with slope m is also m.

The **slopes of perpendicular lines** are opposite reciprocals. For example, if one line has slope $\frac{4}{5}$, then any line perpendicular to it has slope $-\frac{5}{4}$. If a line has slope –3, then any line perpendicular to it has slope $\frac{1}{3}$. In general, the slope of a line perpendicular to a line with slope m is $-\frac{1}{m}$.

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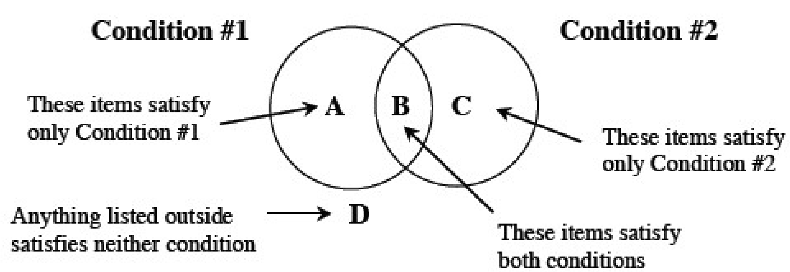
Venn Diagrams

A Venn diagram is a tool used to classify objects. It is usually composed of two or more circles that represent different conditions. An item is placed or represented in the Venn diagram in the appropriate position based on the conditions it meets. See the example below:



METHODS AND MEANINGS

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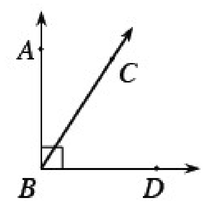
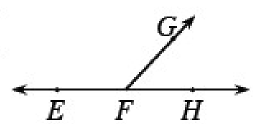
Angle Relationships

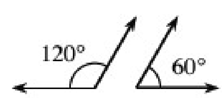
If two angles have measures that add up to 90°, they are called **complementary angles**. For example, in the diagram at right, $\angle ABC$ and $\angle CBD$ are complementary because together they form a right angle.



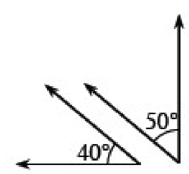
If two angles have measures that add up to 180°, they are called **supplementary angles**. For example, in the diagram at right, $\angle EFG$ and $\angle GFH$ are supplementary because together they form a straight angle.



Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.



Supplementary



Complementary

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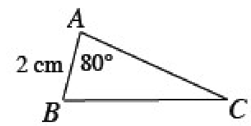
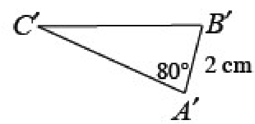
Naming Parts of Shapes

Part of geometry is the study of parts of shapes, such as points, line segments, and angles. To avoid confusion, standard notation is used to name these parts.

A **point** is named using a single capital letter. For example, the vertices (corners) of the triangle at right are named A, B, and C.



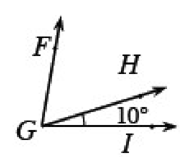
If a shape is transformed, the image shape is often named using **prime notation**. The image of point A is labeled A' (read as "A prime"), the image of B is labeled B' (read as "B prime"), etc. At right, $\Delta A'B'C'$ is the image of ΔABC.



The side of a polygon is a line segment. A **line segment** is a portion of a line between two points and is named by naming its endpoints and placing a bar above them. For example, one side of the first triangle above is named $\overline{AB}$. When referring to the length of a segment, the bar is omitted. In ΔABC above, $AB = 2$ cm.

A **line**, which differs from a segment in that it extends infinitely in either direction, is named by using two points on the line and placing a bar with arrows above them. For example, the line below is named $\overleftrightarrow{DE}$. When naming a segment or line, the order of the letters is unimportant. The line below could also be named $\overleftrightarrow{ED}$.

An **angle** can be named by putting an angle symbol in front of the name of the angle's vertex. For example, the angle measuring 80° in ΔABC above is named $\angle A$. Sometimes using a single letter makes it unclear which angle is being referenced. For example, in the diagram at right, it is unclear which angle is referred to by $\angle G$. When this happens, the angle is named with three letters. For example, the angle measuring 10° is called $\angle HGI$ or $\angle IGH$. Note that the name of the vertex must be the second letter in the name; the order of the other two letters is unimportant.



To refer to an angle's measure, an m is placed in front of the angle's name. For example, $m\angle HGI = 10°$ means "the measure of $\angle HGI$ is 10°."

METHODS AND MEANINGS

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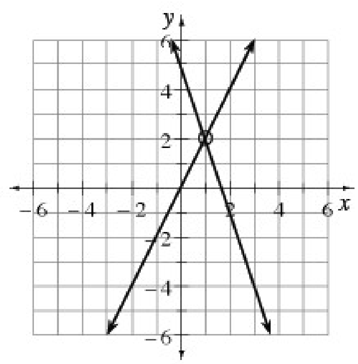
Systems of Linear Equations

In a previous course, you learned that a **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right. In a system, each variable represents the same quantity in both equations. For example, y represents the same quantity in *both* equations at right.

$$y = 2x$$
$$y = -3x + 5$$

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens when the graphs of two lines lie on top of each other. Such lines are said to **coincide**.

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable. For example, to solve the system at right:

$$x = -3y + 1$$
$$4x - 3y = -11$$

- Use substitution to rewrite the two equations as one. In other words, replace x with $(-3y+1)$ to get $4(-3y+1) - 3y = -11$. This equation can then be solved to find y. In this case, $y = 1$.
- To find the point of intersection, substitute to find the other value.
- Substitute $y = 1$ into $x = -3y + 1$ and write the answer for x and y as an ordered pair.
- To test the solution, substitute $x = -2$ and $y = 1$ into $4x - 3y = -11$ to verify that it makes the equation true. Since $4(-2) - 3(1) = -11$, the solution $(-2, 1)$ must be correct.

$$x = -3y + 1$$
$$4(\quad) - 3y = -11$$
$$4(-3y+1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$

$$x = -3(1) + 1 = -2$$
$$(-2, 1)$$

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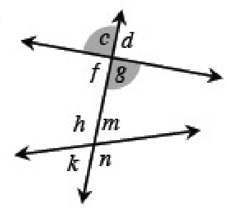
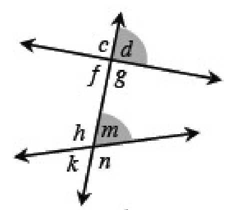
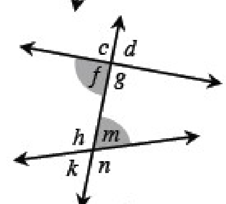
More Angle Pair Relationships



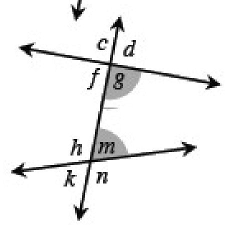
Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles $\angle c$ and $\angle g$ in the diagram at right. $\angle c$ by itself is not a vertical angle, nor is $\angle g$, although $\angle c$ and $\angle g$ together are a pair of vertical angles. Vertical angles always have equal measure.



Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.



$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.



$\angle g$ and $\angle m$ are **same-side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same-side interior angles are supplementary when the lines intersected by the transversal are parallel.

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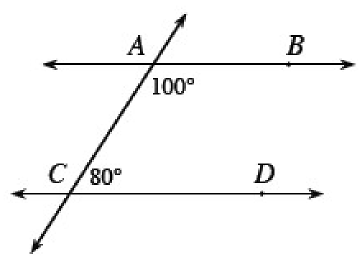
Proof by Contradiction

The kind of argument you used in Lesson 2.1.5 to justify "If same-side interior angles are supplementary, then lines are parallel" is sometimes called a **proof by contradiction**. In a proof by contradiction, you prove a claim by thinking about what the consequences would be if it were false. If the claim's being false would lead to an impossibility, this shows that the claim must be true.

For example, suppose you know Mary's brother is seven years younger than Mary. Can you argue that Mary is at least five years old? A proof by contradiction of this claim would go:

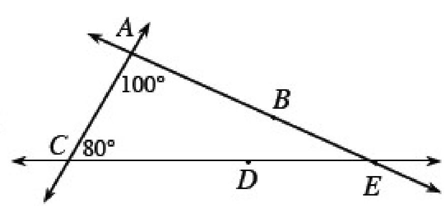
> *Suppose* Mary is less than five years old.
>
> *Then* her brother's age is negative!
>
> *But* this is impossible, so Mary must be at least five years old.

To show that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel in the diagram at right, you used a proof by contradiction. You argued:



Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at some point E.

Then the angles in ΔAEC add up to more than 180°.



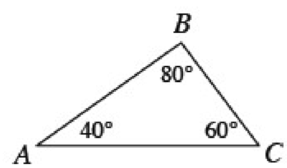
But this is impossible, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel.

This is true no matter on which side of $\overleftrightarrow{AC}$ point E is assumed to be.

METHODS AND MEANINGS

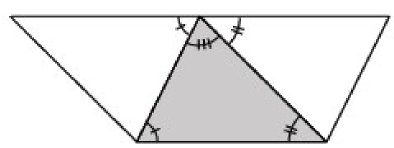
Triangle Angle Sum Theorem

The **Triangle Angle Sum Theorem** states that the measures of the angles in a triangle add up to 180°. For example, in ΔABC at right:



$$m\angle A + m\angle B + m\angle C = 180°$$

The Triangle Angle Sum Theorem can be verified by using a tiling of the given triangle (shaded at right). Because the tiling produces parallel lines, the alternate interior angles must be congruent. As seen in the diagram at right, the three angles of a triangle form a straight angle. Therefore, the sum of the angles of a triangle must be 180°.



MATH NOTES

METHODS AND MEANINGS

Multiplying Binomials

One method for multiplying binomials is to use a generic rectangle. That is, use each factor of the product as a dimension of a rectangle and find its area. If $(2x + 5)$ is the base of a rectangle and $(3x - 1)$ is the height, then the expression $(2x + 5)(3x - 1)$ is the area of the rectangle. See the example below.

	$2x$	$+5$	
-1	$-2x$	-5	-1
$3x$	$6x^2$	$15x$	$3x$
	$2x$	$+5$	

Multiply:

$$
\begin{aligned}
(2x+5)(3x-1) &= 6x^2 - 2x + 15x - 5 \\
&= 6x^2 + 13x - 5
\end{aligned}
$$

2.2.2 Math Notes

METHODS AND MEANINGS

MATH NOTES

Conditional Statements

A **conditional statement** is written in the form "**If …, then ….**" Here are some examples of conditional statements:

If a shape is a rhombus, then it has four sides of equal length.

If it is February 14th, then it is Valentine's Day.

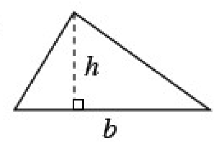
If a shape is a parallelogram, then its area is $A = bh$.

MATH NOTES

METHODS AND MEANINGS

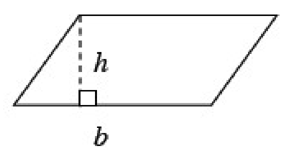
Areas of Triangles, Parallelograms, and Trapezoids

The area of a triangle is half the area of a rectangle with the same base and height. If the base of the triangle is length b and the height length h, then the area of the triangle is:



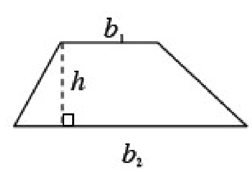
$$A = \frac{1}{2}bh\,.$$

The area of a parallelogram is equal to the area of a rectangle with the same base and height. If the base of the parallelogram is length b and the height length h, then the area of the parallelogram is:



$$A = bh\,.$$

Finally, the area of a trapezoid is found by averaging the lengths of the two bases and multiplying by the height. If the trapezoid has bases b_1 and b_2 and height h, then the area is:



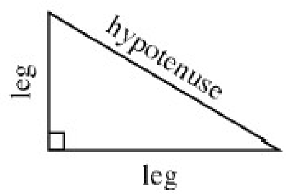
$$A = \frac{1}{2}(b_1 + b_2)h\,.$$

MATH NOTES

METHODS AND MEANINGS

Right Triangle Vocabulary

Several of the triangles that you have been working with in this section are right triangles, that is, triangles that contain a 90° angle. The two shortest sides of the right triangle (the sides that meet at the right angle) are called the **legs** of the triangle and the longest side (the side opposite the right angle) is called the **hypotenuse** of the triangle.



MATH NOTES-Chapters 1-6

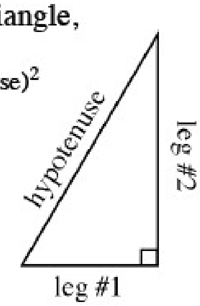
METHODS AND MEANINGS

MATH NOTES

The Pythagorean Theorem

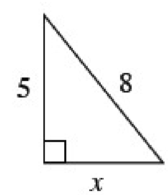
The **Pythagorean Theorem** states that in a right triangle,

$$(\text{length of leg } \#1)^2 + (\text{length of leg } \#2)^2 = (\text{length of hypotenuse})^2$$



The Pythagorean Theorem can be used to find a missing side length in a right triangle. See the example below.

$$5^2 + x^2 = 8^2$$
$$25 + x^2 = 64$$
$$x^2 = 39$$
$$x = \sqrt{39} \approx 6.24$$



In the example above, $\sqrt{39}$ is an example of an **exact** answer, while 6.24 is an **approximate** answer.

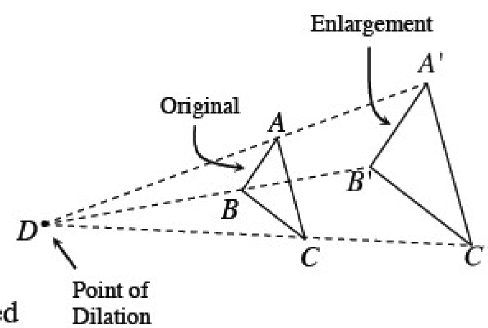
METHODS AND MEANINGS

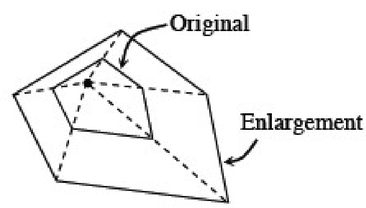
MATH NOTES

Dilations

The transformations you studied in Chapter 1 (translations, rotations, and reflections) are called rigid transformations because they all maintain the size and shape of the original figure.

However, a **dilation** is a transformation that maintains the shape of a figure but multiplies its lengths by a chosen factor. In a dilation, a figure is stretched proportionally from a particular point, called the **point of dilation** or **stretch point**. For example, in the diagram at right, ΔABC is dilated to form $\Delta A'B'C'$. Notice that while a dilation changes the size and location of the original figure, it does not rotate or reflect the original. While lengths can change, angles do not change under a dilation.





Note that if the point of dilation is located inside a shape, the enlargement encloses the original, as shown at right.

METHODS AND MEANINGS

MATH NOTES

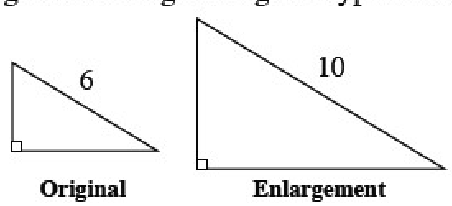
Ratio of Similarity and Zoom Factor

The term ratio was introduced in Chapter 1 in the context of probability. But ratios are very important when comparing two similar figures. Review what you know about ratios below.

A comparison of two quantities (numbers or measures) is called a **ratio**. A ratio can be written as:

$$a:b \quad \text{or} \quad \frac{a}{b} \quad \text{or} \quad a \text{ to } b$$

Each ratio has a numeric value that can be expressed as a fraction or a decimal. For the two similar right triangles below, the ratio of the small triangle's hypotenuse to the large triangle's hypotenuse is $\frac{6}{10}$ or $\frac{3}{5}$. This means that for every three units of length in the small triangle's hypotenuse, there are five units of length in the large triangle's hypotenuse.



$\frac{6}{10}$ ⟶ means 6 units of length on one hypotenuse compared to 10 units on the other hypotenuse

The ratio between any pair of corresponding sides in similar figures is called the **ratio of similarity**.

When a figure is enlarged or reduced, each side is multiplied (or divided) by the same number. While there are several names for this number, this text will refer to this number as the **zoom factor**. Note that "scale factor" is another commonly used term. To help indicate if the figure was enlarged or reduced, the zoom factor is written as the ratio of the new figure to the original figure. For the two triangles above, the zoom factor is $\frac{10}{6}$ or $\frac{5}{3}$.

METHODS AND MEANINGS

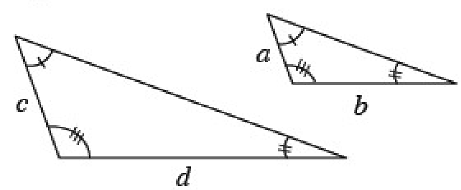
MATH NOTES

Proportional Equations

A **proportional equation** is one that compares two or more ratios. Proportional equations can compare two pairs of corresponding parts (sides) of similar shapes, or can compare two parts of one shape to the corresponding parts of another shape.

For example, the following equations can be written for the similar triangles at right:

$\frac{a}{c} = \frac{b}{d}$ **or** $\frac{a}{b} = \frac{c}{d}$



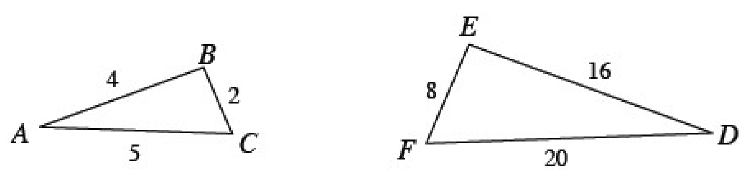
METHODS AND MEANINGS

MATH NOTES

Writing a Similarity Statement

A **similarity transformation** is a sequence of transformations that can include rigid transformations, dilations, or both. Two figures are **similar** if there is a similarity transformation that takes one shape onto the other. Similarity transformations preserve angles, parallelism of two lines, and ratios of side lengths.

To represent the fact that two shapes are similar, use the symbol "~". For example, if there is a similarity transformation that takes ΔABC onto ΔDEF, then you know they are similar and this can be stated as $\Delta ABC \sim \Delta DEF$. The order of the letters in the name of each triangle determines which sides and angles correspond. For example, in the statement $\Delta ABC \sim \Delta DEF$, you can determine that $\angle A$ corresponds to $\angle D$ and that $\overline{BC}$ corresponds to $\overline{EF}$.



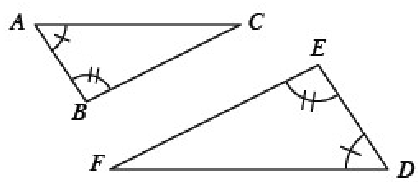
METHODS AND MEANINGS

MATH NOTES

Conditions for Triangle Similarity

When two figures are similar, there is a similarity transformation that takes one onto the other. Since both dilations and rigid motions preserve angles, the corresponding angles must have equal measure. In the same way, since every similarity transformation preserves ratios of lengths, you know that corresponding sides must be proportional.

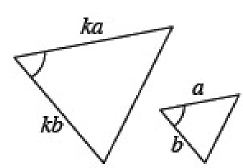
These relationships can also help you decide if two figures are similar. When two pairs of corresponding angles have equal measures, the two triangles must be similar. This is because the third pair of angles must also be equal. This is known as the **AA Triangle Similarity** condition (which can be abbreviated as "AA Similarity" or "AA ~" for short).



AA ~: If two pairs of angles have equal measures, then the triangles are similar.

To prove that AA ~ is true for a pair of triangles with two pairs of congruent angles (like ΔABC and ΔDEF at right), use rigid transformations to move the image A' to point D, the image B' to $\overrightarrow{DE}$ and C' to $\overrightarrow{DF}$. Then you know that $\overleftrightarrow{B'C'}$ and $\overleftrightarrow{EF}$ are parallel because corresponding angles are congruent. This means that the dilation of triangle $\Delta A'B'C'$ from point D to take the image B' to point E will also carry C' to point F. Therefore, $\Delta A'B'C'$ will move onto ΔDEF and you have found a similarity transformation taking ΔABC to ΔDEF.

Another condition that guarantees similarity is referred to as the **SAS Triangle Similarity** condition (which can be abbreviated as "SAS Similarity" or "SAS ~" for short.) The "A" is placed between the two "S"s because the angle is *between* the two sides. Its proof is very similar to the proof for AA ~ above.

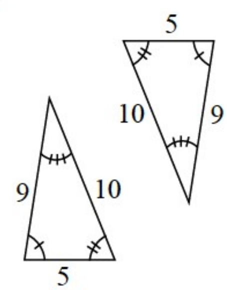


METHODS AND MEANINGS

MATH NOTES

Congruent Shapes

If there is a series of rigid transformations that maps one shape onto the other, then the two shapes are **congruent**. Since the figures must have the same shape, they must be similar.



Two figures are congruent if they meet both of the following conditions:

- The two figures are similar, and
- Their side lengths have a common ratio of 1.

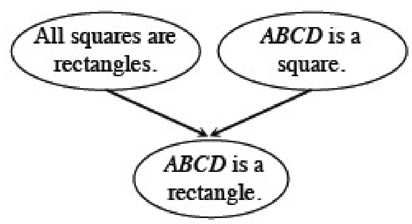
3.2.2 Math Notes

MATH NOTES

METHODS AND MEANINGS

Writing a Flowchart

A **flowchart** helps to organize facts and indicate which facts lead to a conclusion. The bubbles contain facts, while the arrows point to a conclusion that can be made from a fact or multiple facts.



For example, in the flowchart at right, two independent (unconnected) facts are stated: "*All squares are rectangles*" and "*ABCD is a square*." These facts together lead to the conclusion that *ABCD* must be a rectangle. Note that the arrows point toward the conclusion.

3.2.4 Math Notes

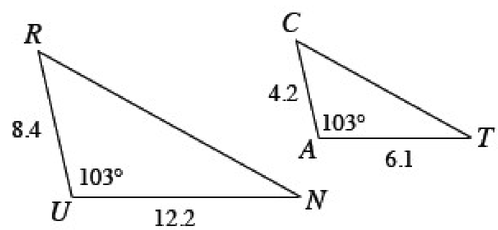
MATH NOTES

METHODS AND MEANINGS

Complete Conditions for Triangle Similarity

There are exactly three valid, non-redundant triangle similarity conditions that use only sides and angles. (A similarity condition is "redundant" if it includes more information than is necessary to establish triangle similarity.) They are abbreviated as: SSS ~, AA ~, and SAS ~. In the SAS~ condition, the "A" is placed between the two "S"s to indicate that the angle must be *between* the two sides used.

For example, ΔRUN and ΔCAT at right are similar by SAS ~. $\frac{RU}{CA} = 2$ and $\frac{UN}{AT} = 2$, so the ratios of the side lengths of the two pairs of corresponding sides are equal. The measure of the angle *between* $\overline{RU}$ and $\overline{UN}$, $\angle U$, equals the measure of the angle between $\overline{CA}$ and $\overline{AT}$, $\angle A$, so the conditions for SAS ~ are met.



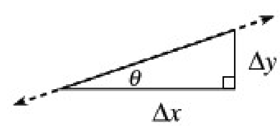
MATH NOTES

METHODS AND MEANINGS

Slope and Angle Notation

The slope of a line is the ratio of the vertical distance to the horizontal distance in a slope triangle formed by two points on a line. The vertical part of the triangle is called Δy, (read "change in y"), while the horizontal part of the triangle is called Δx (read "change in x"). Slope can then be written as $\frac{\Delta y}{\Delta x}$. Slope indicates both how steep the line is and its direction, upward or downward, left to right.

When a side length in a triangle is missing, that length is often assigned a variable from the English alphabet such as x, y, or z. However, sometimes you need to distinguish between an unknown side length and an unknown angle measure. With that in mind, mathematicians sometimes use Greek letters as variables for angle measurement. The most common variable for an angle is the Greek letter θ (*theta*), pronounced "THAY-tah." Two other Greek letters commonly used include α (*alpha*), and β (*beta*), pronounced "BAY-tah."



When a right triangle is oriented like a slope triangle, such as the one in the diagram above, the angle the line makes with the horizontal side of the triangle is called a **slope angle**.

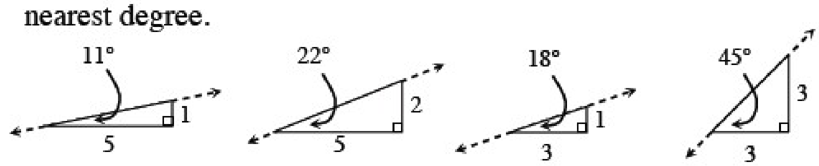
MATH NOTES-Chapters 1-6

METHODS AND MEANINGS

MATH NOTES

Slope Ratios and Angles

In Lesson 4.1.1, you discovered that certain slope angles produce slope triangles with special ratios. Below are the triangles you have studied so far. Note that the angles below are rounded to the nearest degree.



4.1.2 Math Notes

METHODS AND MEANINGS

MATH NOTES

Sequences

A sequence is a function in which the independent variable is a positive integer (usually called the "term number") and the dependent value is the term value. A sequence is usually written as a list of numbers.

Arithmetic Sequences
In an arithmetic sequence, the **common difference** between terms is constant. For example, in the arithmetic sequence 4, 7, 10, 13, …, the common difference is 3.

The equation for an arithmetic sequence is: $t(n) = mn + b$ or $a_n = mn + a_0$ where n is the term number, m is the common difference, and b or a_0 is the zeroth term. Compare these equations to a continuous linear function $f(x) = mx + b$ where m is the growth (slope) and b is the starting value (y-intercept).

For example, the arithmetic sequence 4, 7, 10, 13, … could be represented by $t(n) = 3n + 1$ or by $a_n = 3n + 1$. (Note that "4" is the first term of this sequence, so "1" is the zeroth term.)

Another way to write the equation of an arithmetic sequence is by using the first term in the equation, as in $a_n = m(n-1) + a_1$, where a_1 is the first term. The sequence in the example could be represented by $a_n = 3(n-1) + 4$.

You could even write an equation using any other term in the sequence. The equation using the fourth term in the example would be $a_n = 3(n-4) + 13$.

Geometric Sequences
In a geometric sequence, the **common ratio** or **multiplier** between terms is constant. For example, in the geometric sequence 6, 18, 54, …, the multiplier is 3. In the geometric sequence 32, 8, 2, $\frac{1}{2}$, …, the common multiplier is $\frac{1}{4}$.

The equation for a geometric sequence is: $t(n) = ab^n$ or $a_n = a_0 \cdot b^n$ where n is the term number, b is the sequence generator (the multiplier or common ratio), and a or a_0 is the zeroth term. Compare these equations to a continuous exponential function $f(x) = ab^x$ where b is the growth (multiplier) and a is the starting value (y-intercept).

For example, the geometric sequence 6, 18, 54, … could be represented by $t(n) = 2 \cdot 3^n$ or by $a_n = 2 \cdot 3^n$.

You can write a first term form of the equation for a geometric sequence as well: $a_n = a_1 \cdot b^{n-1}$. For the example, first term form would be $a_n = 6 \cdot 3^{n-1}$.

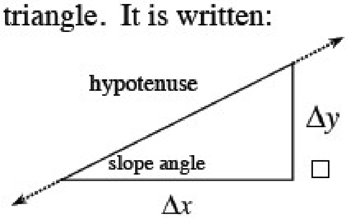
MATH NOTES

METHODS AND MEANINGS

The Tangent Ratio

For any slope angle in a slope triangle, the ratio that compares the Δy to Δx is called the **tangent ratio**. The ratio for any angle is constant, regardless of the size of the triangle. It is written:

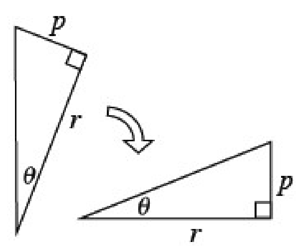
$$\tan(\text{slope angle}) = \frac{\Delta y}{\Delta x}$$



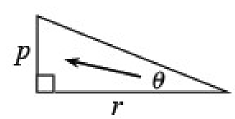
One way to identify which side is Δy and Δx is to first reorient the triangle so that it looks like a slope triangle, as shown at right.

For example, when the triangle at right is rotated, the resulting slope triangle helps to show that the tangent of θ is $\frac{p}{r}$, since θ is the slope angle, p is Δy and r is Δx. This is written:

$$\tan\ \theta = \frac{p}{r}$$



Whether the triangle is oriented as a slope triangle or not, you can identify Δy as the leg that is always opposite (across the triangle from) the angle, while Δx is the leg closest to the angle.



$$\tan\ \theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{p}{r}$$

METHODS AND MEANINGS

MATH NOTES

Independent Events

Two events are **independent** if knowing that one event occurred does not affect the probability of the other event occurring. For example, one probabilistic situation might be about a vocabulary quiz in science class today with possible outcomes {have a quiz, do not have a quiz}. Another probabilistic situation might be the outcome of this weekend's football game with the possibilities {win, lose, tie}. If you know that a quiz occurred today, it does not change the probability of the football team winning this weekend. The two events are independent.

A box contains three red chips and three black chips. If you get a red chip on the first try (and put it back in the box), the probability of getting a red chip on the second try is $\frac{3}{6}$. If you did not get a red chip on the first try, the probability of getting a red chip on the second try is still $\frac{3}{6}$. The probability of getting a red chip on the second try was not changed by knowing whether you got a red chip on the first try or not. When you return the chips to the box, the events {red on first try} and {red on second try} are independent.

However, if an event that occurred changes the probability of another event, the two events are **not independent**. Since getting up late this morning changes the probability that you will eat breakfast, these two events would not be independent.

If you get a red chip on the first try, and *do not put the first chip back in the box*, the probability of a red chip on the second try is $\frac{2}{5}$. If you did not get a red chip on the first try, the probability of getting a red chip on the second try is $\frac{3}{5}$. The probability of getting a red chip on the second try was changed by whether you got a red chip on the first try or not. When you do not replace the chips between draws, the events {red on first try} and {red on second try} are not independent.

Methods and Meanings

Solving a Quadratic Equation

In a previous course, you learned how to solve **quadratic equations** (equations that can be written in the form $ax^2 + bx + c = 0$). Review two methods for solving quadratic equations below.

Some quadratic equations can be solved by **factoring** and using the **Zero Product Property**. For example, because $x^2 - 3x - 10 = (x-5)(x+2)$, the quadratic equation $x^2 - 3x - 10 = 0$ can be rewritten as $(x-5)(x+2) = 0$. The Zero Product Property states that if $ab = 0$, then $a = 0$ or $b = 0$. So, if $(x-5)(x+2) = 0$, then $x - 5 = 0$ or $x + 2 = 0$. Therefore, $x = 5$ or $x = -2$.

Another method for solving quadratic equations is the **Quadratic Formula**. This method is particularly helpful for solving quadratic equations that are difficult or impossible to factor. Before using the Quadratic Formula, the quadratic equation you want to solve must be in standard form, that is, written as $ax^2 + bx + c = 0$.

In this form, a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant term. The Quadratic Formula states:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives two possible answers due to the "$\pm$" symbol. This symbol (read as "plus or minus") is shorthand notation that tells us to calculate the formula twice: once using addition and once using subtraction. Therefore, every Quadratic Formula problem must be simplified twice to give:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

To solve $x^2 - 3x - 10 = 0$ using the Quadratic Formula, substitute $a = 1$, $b = -3$, and $c = -10$ into the formula, as shown below.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} \Rightarrow \frac{3 \pm \sqrt{49}}{2} \Rightarrow \frac{3+7}{2} \text{ or } \frac{3-7}{2} \Rightarrow x = 5 \text{ or } x = -2$$

METHODS AND MEANINGS

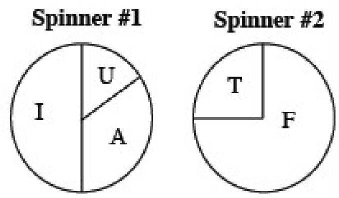
MATH NOTES

Probability Models

When all the possible outcomes of a probabilistic event are *equally likely*, you can calculate probabilities as follows:

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

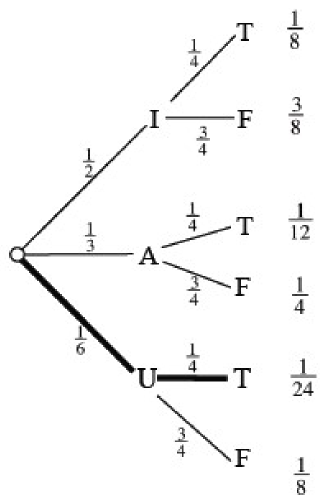
But suppose you spin the two spinners shown to the right. These outcomes are not all equally likely so another model is needed to calculate probabilities of outcomes.



Spinner #2 \ Spinner #1	I $\left(\frac{1}{2}\right)$	A $\left(\frac{1}{3}\right)$	U $\left(\frac{1}{6}\right)$
T $\left(\frac{1}{4}\right)$	IT $\left(\frac{1}{8}\right)$	AT $\left(\frac{1}{12}\right)$	UT $\left(\frac{1}{24}\right)$
F $\left(\frac{3}{4}\right)$	IF $\left(\frac{3}{8}\right)$	AF $\left(\frac{1}{4}\right)$	UF $\left(\frac{1}{8}\right)$

A **probability area model** is practical if there are exactly two probabilistic situations and they are independent. The outcomes of one probabilistic situation are across the top of the table, and the outcomes of the other are on the left. The smaller rectangles are the sample space. Then the probability for an outcome is the area of the rectangle. For example, the probability of spinning "UT" is $\frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24}$. Notice that the area (the probability) of the large overall square is 1.

A **tree diagram** can be used even if there are more than two probabilistic situations, and the events can be independent or not. In this model, the ends of the branches indicate outcomes of probabilistic situations, and the branches show the probability of each event. For example, in the tree diagram at right the first branching point represents Spinner #1 with outcomes "I" "A" or "U". The numbers on the branch represent the probability that a letter occurs.



The numbers at the far right of the table represent the probabilities of various outcomes. For example, the probability of spinning "U" and "T" can be found at the end of the bold branch of the tree. This probability, $\frac{1}{24}$, can be found by multiplying the fractions that appear on the bold branches.

MATH NOTES

METHODS AND MEANINGS

Unions, Intersections, and Complements

A smaller set of outcomes from a sample space is called an **event**. For example, if you draw one card from a standard deck of 52 cards, the sample space would be {A♠, A♣, A♥, A♦, 2♠, 2♣, 2♥, 2♦, … , K♠, K♣, K♥, K♦}. An event might be {drawing a spade}, which would be set {A♠, 2♠, 3♠, … , Q♠, K♠}. The event {drawing a face card} is the set {J♠, J♣, J♥, J♦, Q♠, Q♣, Q♥, Q♦, K♠, K♣, K♥, K♦}.

The **complement** of an event is all the outcomes in the sample space that are not in the original event. For example, the complement of {drawing a spade} would be all the hearts, diamonds, and clubs, represented as the complement of {drawing a spade} = {A♥, 2♥, 3♥, …, Q♥, K♥, A♦, 2♦, 3♦, … , Q♦, K♦, A♣, 2♣, 3♣, … , Q♣, K♣}.

The **intersection** of two events is the event in which *both* the first event *and* the second event occur. The intersection of the events {drawing a spade} and {face card} would be {J♠, Q♠, K♠} because these three cards are in both the event {drawing a spade} *and* the event {face card}.

The **union** of two events is the event in which the first event *or* the second event (or both) occur. The union of the events {drawing a spade} and a {face card} is {A♠, 2♠, 3♠, 4♠, 5♠, 6♠, 7♠, 8♠, 9♠, 10♠, J♠, Q♠, K♠, J♣, J♥, J♦, Q♣, Q♥, Q♦, K♣, K♥, K♦}. This event has 22 outcomes.

The probability of *equally likely* events can be found by:

$$\text{P(event)} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

The probability of {drawing a spade} or {drawing a face card} is $\frac{22}{52}$ because there are 22 cards in the union and 52 cards in the sample space.

Math Notes box continues on next page →

MATH NOTES-Chapters 1-6

METHODS AND MEANINGS

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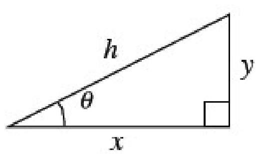
Trigonometric Ratios

You now have three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any right triangle. In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.

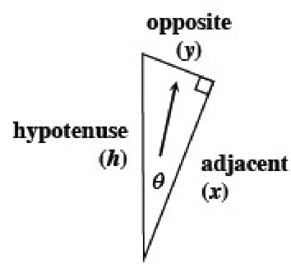


$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

In some cases, you may want to rotate the triangle so that it looks like a slope triangle in order to easily identify the reference angle θ, the opposite leg y, the adjacent leg x, and the hypotenuse h. Instead of rotating the triangle, some people identify the opposite leg as the leg that is always opposite (not touching) the angle. For example, in the diagram at right, y is the leg opposite angle θ.



METHODS AND MEANINGS

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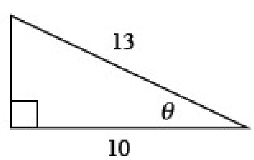
Inverse Trigonometry

Just as subtraction "undoes" addition and multiplication "undoes" division, the inverse trigonometric functions "undo" the trigonometric functions tangent, sine, and cosine. Specifically, **inverse trigonometric functions** are used to find the measure of an acute angle in a right triangle when a ratio of two sides is known. This is the **inverse**, or opposite, of finding the trigonometric ratio from a known angle.

The inverse trigonometric functions that will be used in this course are $\mathbf{sin^{-1}}$, $\mathbf{cos^{-1}}$, and $\mathbf{tan^{-1}}$ (pronounced "inverse sine," "inverse cosine," and "inverse tangent"). Below is an example that shows how $\cos^{-1}$ may be used to find a missing angle, θ.



$$\cos\theta = \frac{10}{13}$$
$$\theta = \cos^{-1}\left(\frac{10}{13}\right)$$
$$\theta \approx 39.7^\circ$$

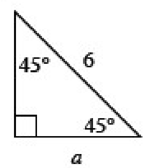
To evaluate $\cos^{-1}\left(\frac{10}{13}\right)$ on a scientific calculator, most calculators require the "2nd" or "INV" button to be pressed before the "cos" button.

MATH NOTES

METHODS AND MEANINGS

Rationalizing a Denominator

In Lesson 5.2.1, you developed some shortcuts to help find the lengths of the sides of a 30°- 60°- 90° and 45°- 45°- 90° triangle. This will enable you to solve similar problems in the future without a calculator or your Trig Table Toolkit.



However, sometimes using the shortcuts leads to some strange looking answers. For example, when finding the length of a in the triangle at right, you will get the expression $\frac{6}{\sqrt{2}}$.

A number with a radical in the denominator is difficult to estimate. Therefore, it is sometimes beneficial to **rationalize the denominator** so that no radical remains in the denominator. Study the example below.

Example: Simplify $\frac{6}{\sqrt{2}}$.

First, multiply the numerator and denominator by the radical in the denominator. Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the expression.

<u>Example</u>

$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2}$$

After multiplying, notice that the denominator no longer has a radical, since $\sqrt{2} \cdot \sqrt{2} = 2$.

Often, the product can be further simplified. Since 2 divides evenly into 6, the expression $\frac{6\sqrt{2}}{2}$ can be rewritten as $3\sqrt{2}$.

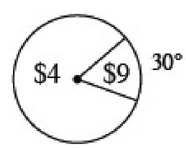
MATH NOTES

METHODS AND MEANINGS

Expected Value

The amount you would expect to win (or lose) per game after playing a game of chance many times is called the **expected value**. This value does not need to be a possible outcome of a single game, but instead reflects an average amount that will be won or lost per game.

For example, the "$9" portion of the spinner at right makes up $\frac{30°}{360°}=\frac{1}{12}$ of the spinner, while the "$4" portion is the rest, or $\frac{11}{12}$, of the spinner. If the spinner was spun 12 times, probability predicts that it would land on "$9" once and "$4" eleven times. Therefore, someone spinning 12 times would expect to receive $1(\$9)+11(\$4)=\$53$. On average, each spin would earn an expected value of $\frac{\$53}{12 \text{ spins}} \approx \$4.42$ per spin. You could use this value to predict the result for any number of spins. For example, if you play 30 times, you would expect to win 30($4.42) = $132.50.



Another way to calculate expected value involves the probability of each possible outcome. Since "$9" is expected $\frac{1}{12}$ of the time, and "$4" is expected $\frac{11}{12}$ of the time, then the expected value can be calculated with the expression $(\$9)(\frac{1}{12})+(\$4)(\frac{11}{12})=\frac{\$53}{12}\approx \$4.42$.

A **fair game** is one in which the expected value is zero. Neither player expects to win or lose if the game is played numerous times.

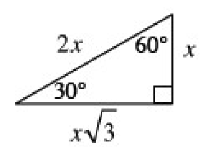
Methods and Meanings

MATH NOTES

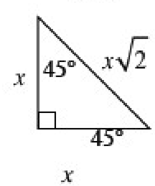
Special Right Triangles

So far in this chapter, you have learned about several special right triangles. Being able to recognize these triangles will enable you to quickly find the lengths of the sides and will save you time and effort in the future.

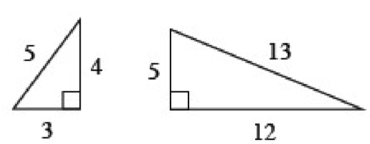
The half-equilateral triangle is also known as the **30°- 60°- 90° triangle**. The sides of this triangle are always in the ratio $1:\sqrt{3}:2$, as shown at right.



Another special triangle is the **45°- 45°- 90° triangle**. This triangle is also commonly known as an isosceles right triangle. The ratio of the sides of this triangle is always $1:1:\sqrt{2}$.



You also discovered several **Pythagorean Triples**. A Pythagorean Triple is any set of 3 positive integers a, b, and c for which $a^2+b^2=c^2$. Two of the common Pythagorean Triples that you will see throughout this course are shown at right.



MATH NOTES

METHODS AND MEANINGS

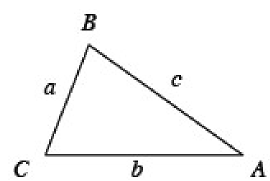
Law of Sines

For any ΔABC, the ratio of the sine of an angle to the length of the side opposite the angle is constant. This means that:

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b},$$

$$\frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}, \text{ and}$$

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle C)}{c}.$$



This property is called the **Law of Sines**. This is a powerful tool because you can use the sine ratio to solve for measures of angles and lengths of sides of *any* triangle, not just right triangles. The law works for angle measures between 0° and 180°.

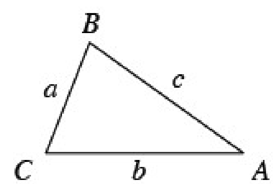
METHODS AND MEANINGS

MATH NOTES

Law of Cosines

Just like the Law of Sines, the **Law of Cosines** represents a relationship between the sides and angles of a triangle.

Specifically, when given the lengths of any two sides, such as a and b, and the angle between them, $\angle C$, the length of the third side, in this case c, can be found using this relationship:



$$c^2 = a^2 + b^2 - 2ab\cos C$$

Similar equations can be used to solve for a and b.

5.3.3 Math Notes

MATH NOTES-Chapters 1-6

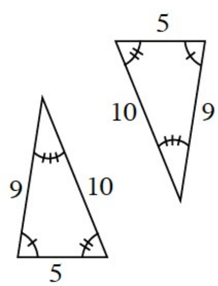
METHODS AND MEANINGS

Congruent Shapes

The information below is from Chapter 3 and is reprinted here for your convenience.

Two figures are **congruent** if there is a sequence of rigid transformations that carries one onto the other. Two figures are also congruent if they meet both of the following conditions:

- The two figures are similar, and
- Their side lengths have a common ratio of 1.



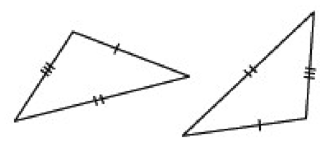
MATH NOTES

METHODS AND MEANINGS

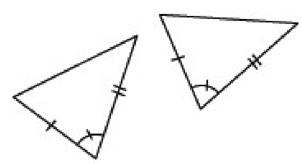
Triangle Congruence Conditions

To show that triangles are congruent, you can show that the triangles are similar and that the common ratio between side lengths is 1 or you can use rigid motions (transformations). However, you can also use certain combinations of congruent, corresponding parts as shortcuts to determine if triangles are congruent. These combinations, called **triangle congruence conditions**, are:

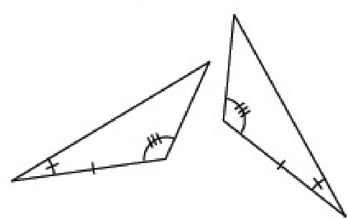
SSS ≅ (Pronounced "side–side–side")
If all three pairs of corresponding sides have equal lengths, then the triangles are congruent.



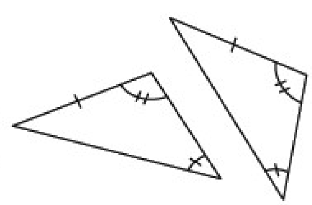
SAS ≅ (Pronounced "side–angle–side")
If two pairs of corresponding sides have equal lengths *and* the angles between them (the included angle) are equal, then the triangles are congruent.



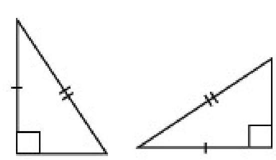
ASA ≅ (Pronounced "angle–side–angle") If two angles and the side between them in a triangle are congruent to the corresponding angles and side in another triangle, then the triangles are congruent.



AAS ≅ (Pronounced "angle–angle–side")
If two pairs of corresponding angles *and* a pair of corresponding sides that are not between them have equal measures, then the triangles are congruent.



HL ≅ (Pronounced "hypotenuse–leg")
If the hypotenuse and a leg of one right triangle have the same lengths as the hypotenuse and a leg of another right triangle, the triangles are congruent.



MATH NOTES

METHODS AND MEANINGS

Converses

When conditional statements (also called "If …, then …" statements) are written backwards so that the condition (the "if" part) is switched with the conclusion (the "then" part), the new statement is called a **converse**. For example, examine the theorem and its converse below:

Theorem: If two parallel lines are cut by a transversal, then pairs of corresponding angles are equal.

Converse: If two corresponding angles formed when two lines are cut by a transversal are equal, then the lines cut by the transversal are parallel.

Since the second statement is a reversal of the first, is called its converse. Note that just because a theorem is true does not mean that its converse must be true. For example, if the conditional statement, "If the dog has a meaty bone, then the dog is happy," is true, but its converse, "If the dog is happy, then the dog has a meaty bone," is not necessarily true. The dog could be happy for other reasons, such as going for a walk.

6.1.5 Math Notes